7. N. N. Yanenko, Method of Fractional Steps in the Solution of Problems of Mathematical Physics [in Russian], Nauka, Novosibirsk (1976).
8. D. A. Nikulin and M. Kh. Strelets, "Possibility of self-oscillatory solutions of nonstationary problems of mixed convection in gas mixtures," Dokl. Akad. Nauk SSSR, 260, No. 3 (1981).
numerical investigation of a gas Jet with heavy particles on the basis
OF A TWO-PARAMETER MODEL OF TURBULENCE
L. B. Gavin, V. A. Naumov,

UDC 532.529
and V. V. Shor

Theoretical [ 1-4] and experimental [5, 6] investigations recently executed show that disperse particles exert substantial influence on the gasdynamic parameters and turbulent structure of two-phase jets. Two fundamental problems occur in the theoretical investigation of flows of this kind: formulation of the initial system of equations and representation of the unknow correlation moments. The solution of the former is obtained in [7] by spatial averaging of the micro-equations describing the processes within the component phases; up to now the 1atter has been solved within the framework of the mixing-path theory [1-3]. The equation of turbulent viscosity transport for a "pure" gas has hence been applied in [2] in place of the Prandtl formula.

Utilized extensively at this time to investigate turbulent flows are two-parameter models containing the energy transport equations of turbulent pulsations and its dissipation velocity [8, 9]. Such models permit not only the computation of the average parameters and characteristics of the turbulence with the stream prehistory taken into account, but also taking account of the influence of the external effects to be given a better foundation. The transport equation of the fluctuating energy was utilized first in [4] to analyze a jet with a low drop concentration under the assumption of no average phase slip. The influence of the drops on the fluctuation energy is here taken into account approximately by the introduction of empirical corrections to the traditional terms describing the turbulent energy generation and dissipation; the scale of turbulence is considered proportional to the jet width.

An $e-\varepsilon$ model is proposed in this paper for the numerical investigation of a turbulent gas jet with solid particles under conditions of a substantial nonequilibrium in the velocities of the component phases; expressions are obtained for the unknown correlation moments due to the presence of a disperse phase.

## 1. FORMULATION OF THE PROBLEM

The system of equations for the average quantities describing the outflow of a two-phase turbulent jet has the form

$$
\begin{gather*}
\frac{\partial u_{g}}{\partial y}+\frac{1}{y} \frac{\partial}{\partial y}\left(y v_{g}\right)=0 ;  \tag{1.1}\\
\frac{\partial}{\partial x}\left(\rho_{p} u_{p}\right)+\frac{1}{y} \frac{\partial}{\partial y}\left(y\left(\rho_{p} v_{p}+\left\langle\rho_{p}^{\prime} p_{p}^{\prime}\right\rangle\right)\right)=0 ;  \tag{1.2}\\
\rho_{g}\left(u_{g} \frac{\partial u_{g}}{\partial x}+v_{g} \frac{\partial u_{g}}{\partial y}\right)+\frac{1}{y} \frac{\partial}{\partial y}\left(y \rho_{g}\left\langle u_{g}^{\prime} v_{g}^{\prime}\right\rangle\right)=-F_{x} ;  \tag{1.3}\\
\rho_{p} u_{p} \frac{\partial u_{p}}{\partial x}+\left(\rho_{p} v_{p}+\left\langle\rho_{p}^{\prime} v_{p}^{\prime}\right\rangle\right) \frac{\partial u_{p}}{\partial y}+\frac{1}{y} \frac{\partial}{\partial y}\left(y \rho_{p}\left\langle u_{p}^{\prime} v_{p}^{\prime}\right\rangle\right)=F_{x} ;  \tag{1.4}\\
u_{g} \frac{\partial \rho}{\partial x}+v_{g} \frac{\partial e}{\partial y}=\frac{1}{y} \frac{\partial}{\partial y}\left(y \frac{v_{t}}{k_{2}} \frac{\partial e}{\partial y}\right)+v_{t}\left(\frac{\partial u_{g}}{\partial y}\right)^{2}-\varepsilon-\varepsilon_{p} ;  \tag{1.5}\\
u_{g} \frac{\partial \varepsilon}{\partial x}+v_{g} \frac{\partial \varepsilon}{\partial y}=\frac{1}{y} \frac{\partial}{\partial y}\left(y \frac{v_{t}}{k_{3}} \frac{\partial \varepsilon}{\partial y}\right)+k_{4} \frac{\varepsilon}{e} v_{t}\left(\frac{\partial u_{g}}{\partial y}\right)^{2}-k_{5} \frac{\varepsilon^{2}}{e}-\Phi_{p}, \tag{1.6}
\end{gather*}
$$

Kaliningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 62-67, January-February, 1984. Original article submitted March 3, 1982.
where $x$, $y$ are the longitudinal and transverse coordinates; $u$, $v$ are projections of the average velocity component on the $x, y$ axes, respectively; $\rho$ is the distributed density; the subscripts $g$ and $p$ refer, respectively, to the parameters of the lifting and disperse phases; e, $\varepsilon$ are the turbulent fluctuation energy and its dissipation rate

$$
e=\frac{1}{2} \sum_{i}\left\langle u_{g i}^{\prime 2}\right\rangle, \quad \varepsilon=v \sum_{i, j}\left\langle\left(\frac{\partial u_{g i}^{\prime}}{\partial x_{j}}\right)_{i}^{2}\right\rangle ;
$$

$\nu$, $v_{t}$ are the molecular and turbulent kinematic viscosities; $u_{g i}^{\prime}$ is the projection of the fluctuating gas velocity component on the i-th axis; $\mathrm{F}_{\mathrm{X}}$ is the projection of the force of interphasal interaction on the x axis, and as in [2], we assume $\mathrm{v}_{\mathrm{g}}=\mathrm{v}_{\mathrm{p}}=\mathrm{v}$.

To make the system of equations more specific, we refine the representation of the interphasal interaction force. It is shown in [10] that for small volume fractions of the disperse phase, the Archimedes force and the force due to the effect of the apparent masses can be neglected. As analysis performed according to [11, 12] for the particle sizes under consideration shows, the contribution of the Magnus force is insignificant in the absence of initial rotation. Then the interphasal interaction force is determined by the velocity nonequilibrium between the phases and has the form

$$
\begin{equation*}
\mathbf{F}^{*}=\frac{3}{4} c_{f}\left(\mathbf{V}_{g}^{*}-\mathbf{V}_{p}^{*}\right)\left|\mathbf{V}_{g}^{*}-\mathbf{V}_{p}^{*}\right| \rho_{g}^{0} \rho_{p} /\left(\rho_{p}^{0} \delta\right), \tag{1.7}
\end{equation*}
$$

where $\delta$ is the particle diameter, $\rho_{p}^{0}=\rho_{p} / \alpha_{p}$ is the true density, $\alpha_{p}$ is the volume fraction of the disperse phase, and the subscript * refers to the actual values of the quantities. The drag coefficient of the spherical particle is determined, as in [2], from the standard curve by the dependence

$$
c_{f}=24\left(1+0.179 \sqrt{\mathrm{Re}^{*}}+0.013 \mathrm{Re}^{*}\right) / \mathrm{Re}^{*}, \mathrm{Re}^{*}=\left|\mathbf{V}_{g}^{*}-\mathbf{V}_{p}^{*}\right| \delta / v .
$$

Let us replace the actual values of the velocities and distributed, densities in (1.7) by the sum of average and fluctuating components $\mathrm{V}^{*}=\mathrm{V}+\mathrm{V}^{\prime}, \rho_{\mathrm{p}}^{*}+\rho_{\mathrm{p}}+\rho_{\mathrm{p}}^{\prime}$. Taking the average, we obtain for the projections of the fluctuating and average components of the interphasal interaction force

$$
\begin{gather*}
F_{i}^{\prime}=\gamma_{i}\left(\left(u_{g i}^{\prime}-u_{p i}^{\prime}\right) \rho_{p}+\frac{\gamma_{y}}{\gamma_{i}}\left(u_{g i}-u_{p i}\right) \rho_{p}^{\prime}+\left(u_{g i}^{\prime}-u_{p i}^{\prime}\right) \rho_{p}^{\prime}-\left\langle\left(u_{g i}^{\prime}-u_{p i}^{\prime}\right) \rho_{p}^{\prime}\right\rangle\right), \\
F_{i}=\gamma_{y}\left(u_{g i}-u_{p i}\right) \rho_{p}+\gamma_{i}\left\langle\left(u_{g i}^{\prime}-u_{p i}^{\prime}\right) \rho_{p}^{\prime}\right\rangle,  \tag{1.8}\\
\gamma_{x}=\beta(1+0.269 \sqrt{\operatorname{Re}}+0.026 \operatorname{Re}), \quad \beta=18 \rho_{g}^{\prime} v /\left(\delta^{2} \rho_{p}^{0}\right), \\
\gamma_{y}=\beta(1+0.179 \sqrt{\operatorname{Re}}+0.013 \mathrm{Re}), \mathrm{Re}=\left|u_{g}-u_{z}\right| \delta / v,
\end{gather*}
$$

Equations (1.5) and (1.6) are obtained from the equations of motion of the lifting medium in whose right side the interphasal interaction force is contained. The additional terms due to the influence of the particles here have the form

$$
\begin{equation*}
\varepsilon_{p}=\frac{1}{\rho_{g}} \sum_{i}\left\langle F_{i}^{\prime} u_{g i}^{\prime}\right\rangle, \quad \Phi_{\nu}=\frac{v}{p_{g}} \sum_{i, j}\left\langle\left\langle\frac{\partial F_{i}^{\prime}}{\partial x_{j}} \frac{\partial u_{g i}^{\prime}}{\partial x_{i}}\right\rangle .\right. \tag{1.9}
\end{equation*}
$$

The boundary conditions are

$$
\begin{align*}
& y=0: \partial u_{g} / \partial y=\partial u_{p} / \partial y=\partial \rho_{p} / \partial y=\partial e / \partial y=\partial \varepsilon / \partial y=v=0 ;  \tag{1.10}\\
& y=\infty: u_{g}=u_{p}=u_{s}, \rho_{p}=\rho_{p_{s}}, e=e_{s}, \varepsilon=\varepsilon_{s} ;  \tag{1.11}\\
& x=0: u_{g}=u_{g 0}(y), u_{p}=u_{p 0}(y), v=v_{0}(y), \rho_{p}=\rho_{p 0}(y),  \tag{1.12}\\
& \\
& \quad e=e_{0}(y), \varepsilon=\varepsilon_{0}(y) .
\end{align*}
$$

The system of equations (1.1)-(1.6) should be closed.
2. CLOSURE OF the system of equations

We use the customary representation [9] for a gas

$$
\left\langle u_{g}^{\prime} v_{g}^{\prime}\right\rangle=-v_{t} \frac{\partial u_{g}}{\partial y}, \quad v_{t}=k_{1} \frac{e^{2}}{\varepsilon} .
$$

Substituting the value of the force fluctuations determined by means of (1.8) into (1.9), we obtain an expression for $\varepsilon_{p}$

$$
\begin{equation*}
\varepsilon_{p}=\frac{1}{\rho_{g}} \sum_{i} \gamma_{i}\left(\left\langle\left(u_{\bar{\sigma} i}^{\prime}-u_{p i}^{\prime}\right) u_{g i}^{\prime}\right\rangle \rho_{p}+\frac{\gamma_{y}}{\gamma_{i}}\left(u_{g i}-u_{p i}\right)\left\langle u_{g i}^{\prime} \rho_{p}^{i}\right\rangle+\left\langle\left(u_{g i}^{\prime}-u_{p i}^{\prime}\right) u_{g i}^{\prime} \rho_{p}^{\prime}\right\rangle\right) \tag{2.1}
\end{equation*}
$$

Let us omit the triple correlation in (2.1); the second term can also be omitted since ugi = $u_{p i}$ for $i=y, z$ while for $i=x$ the correlation moment <u $\left.u_{i}^{\prime} \rho_{p}^{\prime}\right\rangle$ associated with arbitrary mass transfer of the disperse phase because of gas velocity fluctuations is small. We consequently obtain

$$
\varepsilon_{p}=\sum_{i} \gamma_{i}\left(\left\langle u_{g i}^{\prime 2}\right\rangle-\left\langle u_{p i}^{\prime} u_{g i}^{\prime}\right\rangle\right) \rho_{p} / \rho_{g}
$$

For the representation of the correlation moment 〈upiugi> we write the equation of motion of a single particle in integral form [13]:

$$
\begin{equation*}
u_{p i}^{\prime}(t)=u_{p i}^{\prime}(0) \exp \left(-\gamma_{i} t\right)+\gamma_{i} \int_{0}^{t} \exp \left(-\gamma_{i}(t-\tau)\right) u_{g i}^{\prime}\left(r_{0}(\tau), \tau\right) d \tau \tag{2.2}
\end{equation*}
$$

where $u_{g i}^{\prime}\left(r_{p}(\tau), \tau\right)$ is the Euler velocity of the lifting medium along the particle trajectory, and $r_{p}(\tau)$ is the particle displacement. We multiply both sides of (2.2) by ugi( $t$ ) and by taking the average over the ensemble of particles and over time, we obtain

$$
\begin{equation*}
\left\langle u_{p i}^{\prime}(t) u_{g i}^{\prime}(t)\right\rangle=\left\langle u_{q i}^{\prime}(0) u_{g i}^{\prime}(t)\right\rangle \exp \left(-\gamma_{i} t\right)+\gamma_{i} \int_{0}^{t} \exp \left(-\gamma_{i} s\right)\left\langle u_{g i}^{\prime}(\tau) u_{g i}^{\prime}(\tau+s)\right\rangle d s \tag{2.3}
\end{equation*}
$$

The coefficient of Euler space-time correlation of the gas velocity along the particle trajectory will be approximated, as in [13], by the exponential dependence

$$
\begin{equation*}
R_{i j}(s)=\frac{\left\langle u_{g i}^{\prime}(t) u_{g j}^{\prime}(t+s)\right\rangle}{\left\langle u_{g i}^{\prime}(t) u_{g j}^{\prime}(t)\right\rangle}=\exp \left(-\varphi_{i j}|s|\right) \tag{2.4}
\end{equation*}
$$

For the longitudinal and transverse correlation there has been obtained in [13]:

$$
\begin{gathered}
\varphi_{x x}=\left(c_{\beta}\left\langle u_{g}^{\prime 2}\right\rangle^{1 / 2}+\left|u_{g}-u_{p}\right|\right) / \Lambda, \\
\varphi_{y y}=\frac{\left(c_{\beta}\left\langle u_{g}^{\prime 2}\right\rangle^{1 / 2}+\left|u_{g}-u_{p}\right|\right)^{2}}{\left(c_{\beta}\left\langle u_{g}^{\prime 2}\right\rangle^{1 / 2}+0,5\left|u_{g}-u_{p}\right|\right) \Lambda}, \quad c_{\beta} \simeq 1 .
\end{gathered}
$$

The Euler integral spacial scale of turbulence is found from the formula $\Lambda=k_{6} \mathrm{e}^{3 / 2} / \varepsilon$. Integrating (2.3) with (2.4) taken into account for sufficiently large $t$ by using the hypothesis of local homogeneity and isotropy yields

$$
\left\langle u_{p i}^{\prime} u_{g i}^{\prime}\right\rangle=\frac{\gamma_{i}}{\gamma_{i}+\varphi_{i i}}\left\langle u_{g i}^{\prime 2}\right\rangle, \quad \varepsilon_{\vartheta}=\frac{2}{3} e \frac{\rho_{g}}{\rho_{g}} \sum_{i} \frac{\gamma_{i} \dot{\varphi}_{i i}}{\gamma_{i}+\varphi_{i i}}
$$

We set $\Phi_{\mathrm{p}} \simeq 2 \gamma_{\mathrm{x}} \varepsilon \rho_{\mathrm{p}} / \rho_{\mathrm{g}}$.
To obtain the correlation moment $\left\langle u_{p}^{\prime} v_{p}^{\prime}\right\rangle$ we multiply (2.2) for $i=x$ and $i=y$ and after taking the average analogously to [14], we obtain

$$
\begin{equation*}
\left\langle u_{p}^{\prime} v_{p}^{\prime}\right\rangle=\gamma_{x} \gamma_{y} \int_{0}^{\infty} \int_{-\infty}^{\infty} \exp \left(-\left(\gamma_{x}+\gamma_{v}\right) s-\gamma_{x} \tau\right)\left\langle u_{g}^{\prime}(t) v_{g}^{\prime}(t+s)\right\rangle d s d \tau \tag{2.5}
\end{equation*}
$$

Integrating (2.5) with (2.4) taken into account yields

$$
\left\langle u_{p}^{\prime} v_{p}^{\prime}\right\rangle=\frac{\gamma_{x} \gamma_{y}}{\gamma_{x}+\gamma_{y}}\left(\frac{1}{\varphi_{x y}+\gamma_{x}}+\frac{1}{\varphi_{x y}+\gamma_{y}}\right)\left\langle u_{g}^{\prime} v_{g}^{\prime}\right\rangle_{0}
$$

In a first approximation we set $\varphi_{x y}=k_{7} \varphi_{x x^{*}}$.
The correlation moment representing the turbulent mass transfer of the disperse phase can be represented in the form


Fig. 1


Fig. 2


Fig. 3

$$
\left\langle\rho_{p}^{\prime} v_{p}^{\prime}\right\rangle=-D_{p} \frac{\partial \rho_{p}}{\partial y}
$$

The coefficient of transverse particle diffusion is determined, as in [13], from the rms displacement

$$
D_{p}=\frac{1}{2} \lim _{t \rightarrow \infty} \frac{d}{d t}\left\langle y_{p}^{2}(t)\right\rangle=\frac{\left\langle v_{g}^{\prime 2}\right\rangle}{\varphi_{y y}}
$$

## 3. ANALYSIS OF NUMERICAL INVESTIGATION RESULTS

The system of equations (1.1)-(1.6) with the boundary conditions (1.10)-(1.12) closed in this manner was solved by a finite-difference method by using an implicit six-point scheme of second-order accuracy [15]. The conditions for the outflow were taken as in [6] in performing the computations: $\rho_{p}^{0}=8500 \mathrm{~kg} / \mathrm{m}^{3}$, the nozzle radius is $\mathrm{b}=0.015 \mathrm{~m}, \mathrm{u}_{\mathrm{gz}}=\mathrm{u}_{\mathrm{pz}}=35$ $\mathrm{m} / \mathrm{sec}$, the subscript $z$ refers to parameters at the nozzle exit on the jet axis. The submerged jet was considered as a particular case of a cojet for a co-coefficient tending to zero. The co-coefficient did not exceed $1 \%$ in the computations. The values of the empirical constants that remain in (1.3), (1.5), (1.6) as the disperse phase concentration tends to zero taken the same as in single-phase jets [9]. The remaining constants are obtained from the condition of best agreement between the results of computation of two-phase jets and experimental results: $k_{6}=1, k_{7}=2$.

Turbulent fluctuation intensity profiles in the section $x / b=40$ are presented in Fig. 1 for $\delta=45 \cdot 10^{-6} \mathrm{~m}$ for different initial specific particle flow rates $\left(x=\rho_{p} u_{p} / \rho_{g} u_{g}\right)$ : $x_{z}=$ $0 ; 0.25 ; 0.5 ; 1$ are in curves $1-4$, respectively. An increase in $x_{z}$ results in growth of $\varepsilon_{p}, \Phi_{p}$ and the production of turbulent energy. Consequently, as the numerical investigations show, the level of $e$ is reduced somewhat. At the same time the value of the gas velocity on the jet axis rises significantly, consequently the quantity e/ugm is diminished noticeably, which is noted in [1] also; the subscript $m$ refers to parameters on the jet axis. The change in $\mathrm{e} / \mathrm{ugm}_{\mathrm{gm}}^{2}$ is shown in Fig. 2 for $x_{z}=1, x / b=40$ and different particle sizes $\left(10^{-6}\right.$ $\mathrm{m}): 1)$ single-phase jet; 2) $\delta=67$; 3) $\delta=45$; the dashed line is the result of a computation for $\delta=67$ in the absence of the terms $\varepsilon_{p}$, $\Phi_{p}$ in the appropriate equations. A diminution in the particle size causes an increase in $\varepsilon_{p}$, $\Phi_{p}$ because of the growth of $\beta$ and in $v_{t}\left(\partial u_{g} / \partial y\right)^{2}$ because of the growth of $u_{g m}$. It is seen that the turbulent fluctuation intensity in the result diminishes substantially; the additional terms $\varepsilon_{p}$, $\Phi_{p}$ exert substantial influence on the computation results.

A comparison between the results of the numerical computations of the gasdynamic quantity fields (solid lines) and the experimental data [6] is presented in Figs. 3-5. Profiles of the distributed disperse phase density and the longitudinal velocity of both phases along the jet axis are represented in Fig. 3 for $\delta=45 \cdot 10^{-6} \mathrm{~m}$ and different specific particle flow rates: $\rho_{\mathrm{pm}} / \rho_{\mathrm{pz}}[1) \quad x_{z}=1$; 2) $\left.x_{z}=0.5\right]$; ugm/ugz [3) single-phase jet; 4) $x_{z}=0.5$; 5) $\left.x_{z}=1\right] ; u_{\mathrm{pm}} / \mathrm{u}_{\mathrm{pz}}[6) x_{z}=0.5$; 7) $\left.x_{z}=1\right]$. It is seen that an increase in $x_{z}$ causes a


diminuation in the drop of the gas velocity along the jet axis, which is explained not only by the increase in the total particle surface and the magnification of the interphasal interaction, but also by the growth in $\varepsilon_{p}$ noted above. The coefficient of transverse particle diffusion increases here and $\rho_{\mathrm{pm}} / \rho_{\mathrm{pz}}$ diminishes.

Distributions of the same quantities as in Fig. 3 are in Fig. 4 for $\boldsymbol{x}_{\boldsymbol{z}}=1$ and different particle sizes ( $10^{-6} \mathrm{~m}$ ) : $\rho_{\mathrm{pm}} / \rho_{\mathrm{pz}}[1) \delta=45,2$ ) $\left.\delta=67\right]$; $\mathrm{u}_{\mathrm{gm}} / \mathrm{u}_{\mathrm{gz}}$ [3) single-phase jet; 4) $\delta=67$; 5) $\delta=45]$; $\left.\left.u_{\mathrm{pm}} / \mathrm{u}_{\mathrm{pz}}[6) \delta=45 ; 7\right) \delta=67\right]$. An increase in the particle size magnifies the flow velocity nonequilibrium because of a diminution in the interphasal surface. This causes a diminution in $D_{p}$ and an increase in $\rho_{p m} / \rho_{p z}$. A change in the ordinates $y_{p}$, $y_{u}$ in which the disperse phase concentration and the gas velocity, respectively, reach half the axial values is shown in Fig. 5 for $\delta=45 \cdot 10^{-6} \mathrm{~m}$ and different $x_{z}: y_{p}[1) \quad x_{z}=1 ; 2$ ) $x_{z}=$ $0.5]$; $\mathrm{y}_{\mathrm{u}}[3) x_{z}=1$; 4) $x_{z}=0.5$; 5) $\left.x_{z}=0\right]$. As the change increases the jet becomes narrower and longer-range, which agrees with the deductions in [1].

Let us note that because of interaction with the walls the particles in the experiments under consideration [6] visibly acquire a significant initial transverse velocity and the relationship $v_{g}=V_{p}$ is not satisfied. In all probability the turbulent diffusion of the disperse phase mass across the jet is exaggerated; a further refinement of the values for the empirical constant $k_{6}$, on which the quantity $\left\langle\rho_{\mathrm{p}}^{\prime} \mathrm{v}_{\mathrm{p}}^{\prime}\right\rangle$ depends, is necessary.

Better agreement between the results of computations and the experimental data will permit utilization of the model proposed for a numerical investigation of the two-phase turbu1ent jets.

## LITERATURE CITED

1. G. N. Abramovich and T. A. Girshovich, "Turbulent jets carrying solid or liquid drop impurities," Vapor-Liquid Flows [in Russian], Institute of Heat and Mass Transfer, Belorussian Academy of Sciences, Minsk (1977).
2. A. Kartushinskii and F. Frishman, "Numerical analysis of a two-phase turbulent submerged jet," Izv. Akad. Nauk ESSR, Fizika, Matematika, 29, No. 4 (1980).
3. Yu. V. Zuev and I. A. Lepeshinskii, "Mathematical model of a two-phase turbulent jet," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1981).
4. H. Danon, M. Wolfshtein, and G. Hetsroni, "Numerical calculations of two-phase turbulent round jet," Int. J. Mu1tiphase Flow, 3, No. 3 (1977).
5. G. Hetsroni and M. Sokolov, "Distribution of mass, velocity, and intensity of turbulence in a two-phase turbulent jet," J. App1. Mech., 93 (1971).
6. T. A. Girshovich, A. I. Kartushinskii, et al., "Experimental investigation of a turbulent jet carrying heavy impurities," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1981).
7. R. I. Nigmatulin, Principles of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
8. B. A. Kolovandin, Modeling of Heat Transfer for Inhomogeneous Turbulence [in Russian], Nauka i Tekhnika, Minsk (1980).
9. W. Front and T. Maulden (eds.), Turbulence. Principles and Applications [Russian translation], Mir, Moscow (1980).
10. A. I. Ivandaev, A. G. Kutushev, and R. I. Nigmatulin, "Gasdynamics of multiphase media. Shock and detonation waves in gas suspensions," Sci. Eng. Surv. Fluid and Gas Mechanics [in Russian], Vol. 16, VINITI, Moscow (1981).
11. S. Soo, Hydrodynamics of Multiphase Systems [Russian translation], Mir, Moscow (1975).
12. R. Busroyd, Gas Flow with Suspended Particles [Russian translation], Mir, Moscow (1975).
13. A. A. Shraiber, V. N. Milyutin, and V. P. Yatsenko, Hydromechanics of Two-Component Flows with Solid Semidisperse Substance [in Russian], Naukova Dumka, Kiev (1980).
14. L. M. Pismen and A. Nir, "On the motion of suspended particles in stationary homogeneous turbulence," J. Fluid Mech., 84, Pt. 1 (1978).
15. A. L. Dorfman and V. A. Maev, "Numerical modeling of jet viscous fluid flows," Inzh.Fiz. Zh., 31, No. 4 (1976).

INVESTIGATION OF THE GASDYNAMIC PERMEABILITY OF A RETICULAR
FILTER FOR He, Ar, Xe
V. D. Akin'shin, B. T. Porodnov,

UDC 539.217 .082 .3
V. D. Seleznev, and V. V. Surguchev

A number of problems in science, engineering, and technology can be solved with the help of fine-pore membranes. In recent years, membranes which are called reticular filters are finding increasing applications. These membranes are obtained by bombarding thin polymeric films with high-energy ions [1]. The distinguishing features of such membranes are: high density and nonoverlapping of pores, almost cylindrical shape of the channels, and small variance of the pores with respect to radius. All of these properties make reticular filters irreplaceable in many cases, because the quality of the filtration in them is higher than for other types of membranes.

There is a large number of papers in the literature concerning the investigation of gas flows with arbitrary Knudsen numbers through capillaries and capillary sieves with controllable geometry. It is shown in these papers that the gasdynamic permeability of different gases is sensitive to the geometry, material, temperature, and roughness of the surface of the channel [2, 4]. There are practically no analogous investigations of reticular filters of the type indicated above, although they are of undoubted interest, because they permit clarifying both the characteristics of the structure of pores in the reticular filters themselves and the physical nature of the interaction of molecules of different gases with one another and with the surface of channels.

In this paper we present the results of measurements of the flows of helium, argon, and xenon through a reticular filter in a wide range of Knudsen numbers ( $0.5<\mathrm{Kn}<500$ ) at a temperature of $\mathrm{T}=293^{\circ} \mathrm{K}$.

The measurement procedure and the schematic of the setup used to perform the investigations are described in detail in [4]. The method of stationary flow, based on determining the rate of change of the volume of the system at constant pressure $p$, was used. The gas flowing through the reticular filter discharged into a vacuum. The filter under study was prepared from a polyethylene terephthalate $f i 1 \mathrm{~m} L=(1.00 \pm 0.05) \cdot 10^{-3} \mathrm{~cm}$ thick. The pore density of the filter was $N=(4.0 \pm 0.1) \cdot 10^{7} \mathrm{~cm}^{-2}$, the average radius of the input cross sections of the pores was $\left\langle\mathrm{R}_{0}\right\rangle=(1.25 \pm 0.30) \cdot 10^{-5} \mathrm{~cm}$, the average radius of the outlet openings was $\left\langle\mathrm{R}_{\mathrm{L}}\right\rangle=(2.6 \pm 0.3) \cdot 10^{-5} \mathrm{~cm}$. The quantities $N,\left\langle\mathrm{R}_{0}\right\rangle,\left\langle\mathrm{R}_{\mathrm{L}}\right\rangle$ were estimated from photographs of the filter, obtained with the help of an electron microscope.

The experimental results for different gases were compared in terms of the relative flow rate $\omega$ as a function of the rarefaction parameter $\delta$ related to the Knudsen number by the relation

$$
\begin{equation*}
\delta=\frac{\sqrt{\pi}}{2} \frac{\langle R\rangle}{\lambda}=\frac{\sqrt{\pi}}{2} \frac{1}{K n}, \tag{1}
\end{equation*}
$$

Sverdlovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 67-69, January-February, 1984. Original article submitted December 14, 1982.

